

**ADVANCED GCE  
MATHEMATICS (MEI)**

**4754A**

Applications of Advanced Mathematics (C4) Paper A

**QUESTION PAPER**

Candidates answer on the printed answer book.

**OCR supplied materials:**

- Printed answer book 4754A
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Monday 13 June 2011**

**Morning**

**Duration:** 1 hour 30 minutes

**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.**  
Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

**INSTRUCTION TO EXAMS OFFICER / INVIGILATOR**

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

**Section A (36 marks)**

**1** Express  $\frac{1}{(2x+1)(x^2+1)}$  in partial fractions. [5]

**2** Find the first three terms in the binomial expansion of  $\sqrt[3]{1+3x}$  in ascending powers of  $x$ . State the set of values of  $x$  for which the expansion is valid. [5]

**3** Express  $2 \sin \theta - 3 \cos \theta$  in the form  $R \sin(\theta - \alpha)$ , where  $R$  and  $\alpha$  are constants to be determined, and  $0 < \alpha < \frac{1}{2}\pi$ .

Hence write down the greatest and least possible values of  $1 + 2 \sin \theta - 3 \cos \theta$ . [6]

**4** A curve has parametric equations

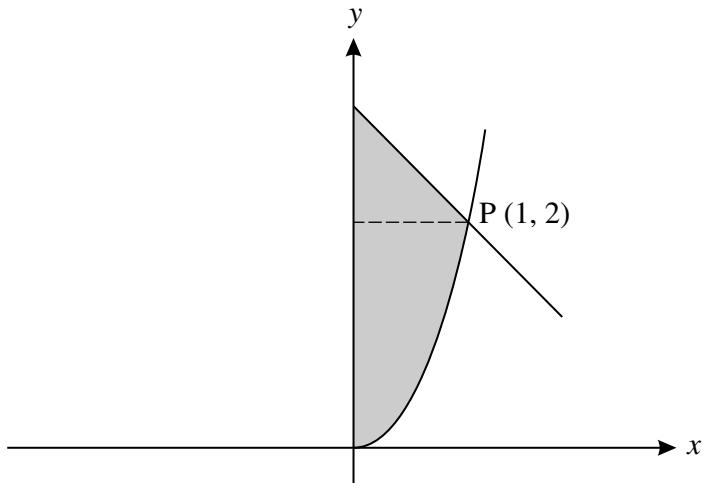
$$x = 2 \sin \theta, \quad y = \cos 2\theta.$$

(i) Find the exact coordinates and the gradient of the curve at the point with parameter  $\theta = \frac{1}{3}\pi$ . [5]

(ii) Find  $y$  in terms of  $x$ . [2]

**5** Solve the equation  $\operatorname{cosec}^2 \theta = 1 + 2 \cot \theta$ , for  $-180^\circ \leq \theta \leq 180^\circ$ . [6]

**6** Fig. 6 shows the region enclosed by part of the curve  $y = 2x^2$ , the straight line  $x + y = 3$ , and the  $y$ -axis. The curve and the straight line meet at P (1, 2).



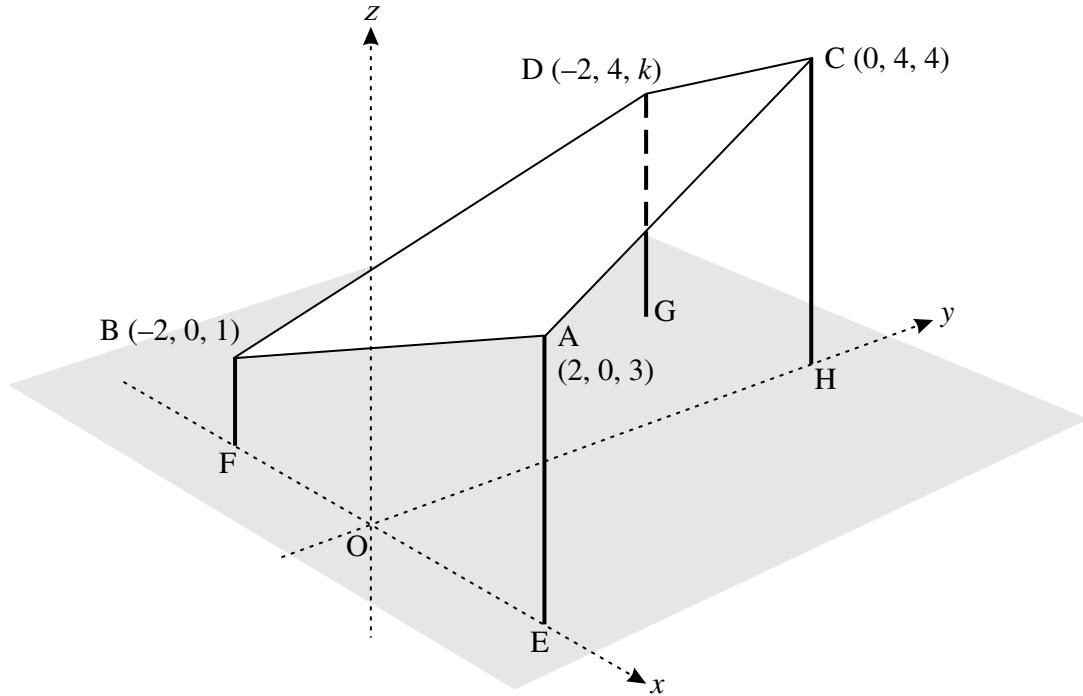
**Fig. 6**

The shaded region is rotated through  $360^\circ$  about the  $y$ -axis. Find, in terms of  $\pi$ , the volume of the solid of revolution formed. [7]

[You may use the formula  $V = \frac{1}{3}\pi r^2 h$  for the volume of a cone.]

**Section B (36 marks)**

- 7 A piece of cloth ABDC is attached to the tops of vertical poles AE, BF, DG and CH, where E, F, G and H are at ground level (see Fig. 7). Coordinates are as shown, with lengths in metres. The length of pole DG is  $k$  metres.

**Fig. 7**

(i) Write down the vectors  $\overrightarrow{AB}$  and  $\overrightarrow{AC}$ . Hence calculate the angle BAC. [6]

(ii) Verify that the equation of the plane ABC is  $x + y - 2z + d = 0$ , where  $d$  is a constant to be determined.

Calculate the acute angle the plane makes with the horizontal plane. [7]

(iii) Given that A, B, D and C are coplanar, show that  $k = 3$ .

Hence show that ABDC is a trapezium, and find the ratio of CD to AB. [5]

**[Question 8 is printed overleaf.]**

- 8 Water is leaking from a container. After  $t$  seconds, the depth of water in the container is  $x$  cm, and the volume of water is  $V$  cm<sup>3</sup>, where  $V = \frac{1}{3}x^3$ . The rate at which water is lost is proportional to  $x$ , so that  $\frac{dV}{dt} = -kx$ , where  $k$  is a constant.

(i) Show that  $x \frac{dx}{dt} = -k$ . [3]

Initially, the depth of water in the container is 10 cm.

(ii) Show by integration that  $x = \sqrt{100 - 2kt}$ . [4]

(iii) Given that the container empties after 50 seconds, find  $k$ . [2]

Once the container is empty, water is poured into it at a constant rate of 1 cm<sup>3</sup> per second. The container continues to lose water as before.

(iv) Show that,  $t$  seconds after starting to pour the water in,  $\frac{dx}{dt} = \frac{1-x}{x^2}$ . [2]

(v) Show that  $\frac{1}{1-x} - x - 1 = \frac{x^2}{1-x}$ .

Hence solve the differential equation in part (iv) to show that

$$t = \ln\left(\frac{1}{1-x}\right) - \frac{1}{2}x^2 - x. \quad [6]$$

(vi) Show that the depth cannot reach 1 cm. [1]

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